

The Significance of the Poynting Vector

*Frederick David Tombe,
Belfast, Northern Ireland,
United Kingdom,
sirius184@hotmail.com
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Abstract. The Poynting vector, $S = E \times H$, represents the rate of flow of electromagnetic energy per unit area per unit time. It appears in Poynting's theorem because of the involvement of Ampère's circuital law and Faraday's law of time-varying electromagnetic induction. It will now be investigated as to whether or not the Poynting vector has any significance if the E field is an electrostatic field, or would it just amount to multiplying apples and bananas?

Historical Background

I. A telegrapher's equation linking the speed of light to electric signals propagating along a conducting wire was first derived by German physicist Gustav Kirchhoff in 1857 [1]. In Kirchhoff's theory, it was assumed that the energy travelled inside the conducting wires. Some years later however, in 1883, English physicist John Henry Poynting made a proposal regarding the transfer of energy in electric circuits. Poynting proposed that at least some of the energy is actually transferred through the space outside the conducting wires [2]. This idea was also taken up around about the same time by English electrical engineer Oliver Heaviside [3].

It was already known since the time of Faraday and Henry that electrical energy can be transferred through the space between two electric circuits in the case of electromagnetic induction, but Poynting and Heaviside were now suggesting that in the case of electrical energy that is applied directly to a circuit, that some of the energy travels through the space in the immediate vicinity of the conducting wires.

Poynting's Theorem

II. The derivation of Poynting's Theorem in this section begins by considering the equation of continuity as applied to the sum of two energy density fields in space. One of these is the electromagnetic energy density field, $\frac{1}{2}[\epsilon_0 \mathbf{E}_K \cdot \mathbf{E}_K + \mu_0 \mathbf{H} \cdot \mathbf{H}]$, which is sourced in a dynamic magnetic field where $\mathbf{E}_K = -\partial \mathbf{A} / \partial t$, and where the magnetic vector potential, \mathbf{A} , satisfies $\nabla \times \mathbf{A} = \mu_0 \mathbf{H}$. The other is the electrostatic energy density field, $\frac{1}{2} \epsilon_0 \mathbf{E}_S \cdot \mathbf{E}_S$, which is sourced in an identifiable electric charge and where $\mathbf{E}_S = -\nabla \phi$. The two $\mathbf{E} \cdot \mathbf{E}$ terms represent the potential energy associated with stress in the all-pervading elastic solid which acts as the medium for the propagation of light. This mathematical form is in the likeness of the form used for the potential energy, $\frac{1}{2} k x^2$, that is stored in a stretched mechanical spring, where \mathbf{E} corresponds to k . See equation (8) in section IV below, and also Part III in Maxwell's 1861 paper "*On Physical Lines of Force*" [4]. The electric permittivity, ϵ_0 , is the inverse of the elastic constant. Meanwhile, the $\mathbf{H} \cdot \mathbf{H}$ term represents fine-grained kinetic energy in the magnetic field in like manner to the familiar mechanical term, $\frac{1}{2} m v^2$, where \mathbf{H} corresponds to the speed, v . See Parts I and II in Maxwell's 1861 paper. The magnetic permeability, μ_0 , is the mass density term. The total energy density, w , is therefore,

$$w = \frac{1}{2} \epsilon_0 \mathbf{E}_K^2 + \frac{1}{2} \mu_0 \mathbf{H}^2 + \frac{1}{2} \epsilon_0 \mathbf{E}_S^2 \quad (1)$$

Taking the partial time derivative of (1) leads to,

$$\partial w / \partial t = \epsilon_0 \mathbf{E}_K \cdot \partial \mathbf{E}_K / \partial t + \mu_0 \mathbf{H} \cdot \partial \mathbf{H} / \partial t + \epsilon_0 \mathbf{E}_S \cdot \partial \mathbf{E}_S / \partial t \quad (2)$$

The first term on the right-hand side of equation (2) contains Maxwell's displacement current, $\epsilon_0 \partial \mathbf{E}_K / \partial t$, as is used in electromagnetic radiation, and we know from Ampère's Circuital Law, as applied in space, that,

$$\nabla \times \mathbf{H} = \epsilon_0 \partial \mathbf{E}_K / \partial t \quad (3)$$

Regarding the second term on the right-hand side of equation (2), we know from Faraday's Law that,

$$\nabla \times \mathbf{E}_K = -\mu_0 \partial \mathbf{H} / \partial t \quad (4)$$

Substituting equations (3) and (4) into equation (2) we get,

$$\partial w / \partial t = \mathbf{E_K} \cdot \nabla \times \mathbf{H} - \mathbf{H} \cdot \nabla \times \mathbf{E_K} + \epsilon_0 \mathbf{E_S} \cdot \partial \mathbf{E_S} / \partial t \quad (5)$$

Hence, using the vector identity,

$$\partial w / \partial t = - \nabla \cdot (\mathbf{E_K} \times \mathbf{H}) + \epsilon_0 \mathbf{E_S} \cdot \partial \mathbf{E_S} / \partial t \quad (6)$$

the Poynting vector, \mathbf{S} , is then defined as,

$$\mathbf{S} = \mathbf{E_K} \times \mathbf{H} \quad (7)$$

The Significance of the Poynting Vector

III. By comparison with the equation for the continuity of charge, the Poynting vector is analogous to electric current density \mathbf{J} , hence it represents the flow of energy per unit area per unit time. One might say that the Poynting vector represents a current of electromagnetic energy which comprises both an electric component and a magnetic component. The question still arises however as to what these two components actually mean in real terms. Reducing it all to the hydrodynamics of the fundamental electric fluid (or aether) from which everything is made, it will be proposed that the electric force term, $\mathbf{E_K}$, represents potential energy, and more specifically hydrostatic aether pressure, while the magnetic term, \mathbf{H} , represents kinetic energy, and more specifically aether flow. Hence the two terms are related to each other through Bernoulli's Principle, and it is proposed that Faraday's law relates to the conversion between pressure and flow in a sea of tiny aether vortices that fills all of space. Transverse pressure, $\mathbf{E_K}$, in a vortex gives way to angular acceleration, $\partial \mathbf{H} / \partial t$, where \mathbf{H} represents the vorticity of the vortex. The rate of flow of the aether, weighted for its hydrostatic pressure would represent the rate of flow of total electromagnetic energy in the same way that electric current density, \mathbf{J} , is the product $\rho \mathbf{v}$.

Electromagnetic radiation would therefore appear to be a complex electric current that flows through space, and when it strikes a conducting wire, the component that strikes the wire at right angles, channels into a simple conduction current, \mathbf{J} , which then flows along the wire. This is like the case of convectively induced electromagnetic induction where a current is induced in a conducting wire that moves at right angles to a magnetic field.

It is proposed that the electromagnetic Poynting vector represents the power density of a complex electric current undergoing a fine-grained vortex flow of electric fluid through a dense sea of rotating electron-

positron dipoles [5], [6], [7], [8]. These dipolar vortices will have a vorticity, \mathbf{H} , equivalent to the magnetic intensity, where $\mathbf{H} = 2\boldsymbol{\omega}$. This current is continually flowing between neighbouring rotating electron-positron dipolar vortices, each which constitutes a tiny electric circuit in its own right. This complex electric current is *Maxwell's Displacement Current*. When emitted from an alternating current source, \mathbf{E}_K and \mathbf{H} will be out of phase by ninety degrees due to Bernoulli's Principle. See **Appendix I**.

The Poynting vector therefore applies to wireless radiation providing that we can isolate \mathbf{H} from that of the already existing background magnetic field [9]. In the case of AC transformers, the Poynting vector would apply to the energy that leaves the primary circuit, flows through space, and enters the secondary circuit.

The Electrostatic Component

IV. The electrostatic component, \mathbf{E}_S , is not included in the Poynting vector as derived and defined in Section **II** above. In J.H. Poynting's original derivation [2], \mathbf{E}_S is present, but it should not have been. Poynting's own derivation begins as if he has ignored \mathbf{E}_K altogether. He starts by splitting the energy terms at equation (1) (*in his own paper*) into electrostatic and electromagnetic energy components. But in the electromagnetic energy component, he uses only \mathbf{H} and he omits \mathbf{E}_K . That is his first mistake. Then at equation (5) (*in his own paper*), he produces Maxwell's electromotive force equation. This was originally equation (77) in Maxwell's 1861 paper [4], and later equation (D) in the original listing of eight Maxwell's equations in his 1865 paper "*A Dynamical Theory of the Electromagnetic Field*" [10]. J.H. Poynting then substitutes all three of Maxwell's EMF terms into what had been exclusively an electrostatic energy component. This meant that he had added the electrostatic term into itself, along with the time-varying electromagnetic term \mathbf{E}_K , and along with the convective electromagnetic term, $\mu_0 \mathbf{v} \times \mathbf{H}$, which ultimately was not involved in the Poynting vector. Next, he segregates both non-convective terms, \mathbf{E}_S and \mathbf{E}_K , from the convective term, and from then on, he treats the two non-convective terms as a single bundled entity.

By comparison with the less cumbersome derivation in Section **II** above, it would be like as if J.H. Poynting had used a single potential energy term in equation (1) in the form $\frac{1}{2} \epsilon_0 (\mathbf{E}_S + \mathbf{E}_K)^2$. This would only be legitimate if \mathbf{E}_S and \mathbf{E}_K were two mutually orthogonal vectors, but in general they are not. The electrostatic field, \mathbf{E}_S , can be superimposed at any angle to the electromagnetic field \mathbf{E}_K . In a transmission line, the two are orthogonal, but this still doesn't legitimize the presence of \mathbf{E}_S in the

Poynting vector, because \mathbf{E}_S is not involved in the constituent equations (3) and (4).

So, what about the electrostatic term, $\epsilon_0 \mathbf{E}_S \cdot \partial \mathbf{E}_S / \partial t$, which is the last term on the right-hand side of equation (2)? It relates to a time-varying electrostatic field, such as we would find in the vicinity of a charging capacitor. The term $\epsilon_0 \mathbf{E}_S \cdot \partial \mathbf{E}_S / \partial t$ relates to linear polarization in a dielectric, where Maxwell's fifth equation, the electric elasticity equation, applies as in,

$$\mathbf{D} = -\epsilon_0 \mathbf{E}_S \quad (8)$$

and hence,

$$\epsilon_0 \mathbf{E}_S \cdot \partial \mathbf{E}_S / \partial t = -\mathbf{E}_S \cdot \mathbf{J}_D \quad (9)$$

where \mathbf{D} is the electric displacement vector and where,

$$\mathbf{J}_D = \partial \mathbf{D} / \partial t \quad (10)$$

is the original displacement current as derived by Maxwell in Part III of his 1861 paper [4], [11], [12]. Although Maxwell originally used dielectric polarization in order to derive displacement current in 1861, when he came to apply it to the derivation of the electromagnetic wave equation in his 1865 paper, [10], he was now using \mathbf{E}_K in the displacement current, [12].

Meanwhile, equation (6) now becomes,

$$\nabla \cdot \mathbf{S} + \mathbf{J}_D \cdot \mathbf{E}_S = -\partial w / \partial t \quad (11)$$

Conclusion - Multiplying Apples and Bananas?

V. Ultimately, the product $\mathbf{E} \times \mathbf{H}$ is a power product similar in principle to the product VI (voltage times current) in circuit electricity. It is a measure of the rate of change of potential energy into kinetic energy in a hydrodynamical system. The \mathbf{E} term is a force term which drives the aethereal electric current that is intrinsic to EM radiation, and hence it also drives the changing magnetic field, $\partial \mathbf{H} / \partial t$.

Poynting's theorem deals with the dynamic state, and the Poynting vector applies to the rate of energy flow in wireless radiation where Faraday's law of time-varying EM induction is involved. The theorem also applies to charging and discharging capacitors, and to linear polarization current in a dielectric.

There is also the convective state to be discussed. A transmission line pulse, [13], involves what would have been a linear polarization field propagating at close to the speed of light* in the space between two conducting wires. But because the pulse is travelling at close to the speed of light, any would-be electrostatic field arising from the pulse's electric charge will have converted, nearly totally, into a magnetic field. And since the pulse is travelling on its own momentum, with no time-varying electromagnetic induction involved, we cannot use a Poynting vector in this context.

Finally, if we were to form the product $\mathbf{E}_s \times \mathbf{H}$ outside of Poynting's theorem in connection with a stationary electrostatic field superimposed upon the magnetic field of a stationary bar magnet, the product would indicate a non-zero value even though there is no actual flow of energy occurring. The electrostatic field in this case is not driving the magnetic field and so we could safely say that it would be a case of multiplying apples and bananas. It would be like multiplying the force of a motorcar engine in London, England, with the speed of a motorcar in Cape Town, South Africa.

* See **Appendix II** – The Speed of Light

Appendix I **(Electromagnetic Waves)**

A diagram in Maxwell's 1873 publication "*A Treatise on Electricity and Magnetism*" [14], indicates that Maxwell believed that the electric displacement, \mathbf{D} , and the magnetic force, \mathbf{H} , in an electromagnetic wave are mutually perpendicular to each other, as well as being in phase with each other in time. On page 389 of the Treatise, under the heading "*Plane Waves*", Maxwell begins the analysis with the magnetic induction equation, $\nabla \times \mathbf{A} = \mathbf{B}$, and he identifies the magnetic induction vector, \mathbf{B} , with *magnetic disturbance*. At equation (14), Maxwell writes Ampère's Circuital Law as $\mu \mathbf{J} = \nabla \times \mathbf{B} = -\nabla^2 \mathbf{A}$, and he identifies the electric current density, \mathbf{J} , with *electric disturbance*. The magnetic disturbance and the electric disturbance will therefore be mutually perpendicular and in time-phase with each other. Equation (15), $\mathbf{J} = \partial \mathbf{D} / \partial t$, where \mathbf{D} is the *electric displacement*, tells us that if \mathbf{J} and \mathbf{D} obey a sinusoidal relationship in time, then they will be out of phase with each other in time by ninety degrees. Since $\mathbf{B} = \mu \mathbf{H}$, where \mathbf{H} is the magnetic force, it follows then that the magnetic force and the electric displacement will be out of phase in time by ninety degrees.

Fig. 66 on page 390 however shows \mathbf{H} and \mathbf{D} to be in time-phase with each other, and so this would appear to be an error. While Maxwell's plane wave solutions at equation (20) in his Treatise were in the electromagnetic momentum, \mathbf{A} , modern textbooks provide similar solutions in \mathbf{H} and \mathbf{E}_K , where $\mathbf{D} = \epsilon\mathbf{E}_K$, and where \mathbf{E}_K is the electromotive force induced by time-varying electromagnetic induction, as per the Maxwell-Faraday equation, $\nabla \times \mathbf{E}_K = -\partial\mathbf{B}/\partial t$. These sinusoidal solutions are used to prove that \mathbf{H} and \mathbf{E}_K are in time-phase with each other. However, these sinusoidal solutions ignore the full three-dimensional physical interrelationships between \mathbf{A} , \mathbf{H} , and \mathbf{E}_K within the context of the vortices through which they were initially defined. They ignore the fact that an EM wave involves a chain reaction of precessing vortices, in which the energy is exchanged between neighbouring vortices when \mathbf{H} is pointing along the direction of wave propagation. See *“Wireless Telegraphy Beyond the Near Magnetic Field”* [9]. The textbook solutions on the other hand only consider the projection of \mathbf{H} perpendicular to the direction of propagation where it appears to have reached its maximum magnitude at the same moment in time when \mathbf{E}_K reaches its maximum magnitude. In actual fact though, \mathbf{H} reaches its absolute maximum magnitude when it has rotated downwards parallel to the direction of propagation.

Appendix II (The Speed of Light)

The correspondence between the speed of an electrical signal along a wire on the one hand, and the speed of light on the other hand, is based largely on aether hydrodynamics, on the principle that electric current is primarily an aethereal fluid that flows between positive particles (sources) and negative particles (sinks), and at an average speed in the order of the speed of light.

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In relation to the speed of light, ***“The most probable surmise or guess at present is that the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves— i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed”***

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